# The Steiner Selection: Geometry, Stability, and Sensitivity

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**Abstract**

The Steiner point is a classical geometric construction that assigns a canonical point to a convex body. It plays a central role in convex geometry and has important applications in set-valued and variational analysis.

In this talk, we study the geometry, stability, and sensitivity of the Steiner selection for parameterized families of convex bodies. We establish stability-transfer principles showing that regularity properties of convex-valued mappings are inherited by their Steiner selections, yielding Lipschitz continuity with respect to Hausdorff perturbations.

We further analyze the differential structure of the Steiner selection. Under mild assumptions, we prove almost-everywhere differentiability and derive an explicit Jacobian formula describing its first-order variation in terms of directional derivatives of the support function. This representation reveals structural properties of the Jacobian, including sparsity and anisotropic behavior, and leads to refined Lipschitz estimates. Under stronger assumptions, a $C^{1,1}$-type regularity is obtained.

In the polyhedral case, the general formulas reduce to finite expressions involving vertex data, allowing a precise description of how variations of the normal fan affect the Steiner point. We also consider smooth deformations of convex bodies and derive first- and second-order variation formulas, highlighting connections with regularization techniques in nonsmooth analysis.